Paper Class – XII

Subject – MATHEMATICS

ANSWER SECTION A AND EITHER SECTION B OR C. ***TIME 3 HRS***

SECTION A (ANSWER Q1. AND ANY FIVE) [10x3+10x5 = 80 marks]

Question 1

1. a) If $A=\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ & $..B=\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and A^2=B, find x. [x=1]

   b) Evaluate : $\int \frac{e^x(1+\tan x)dx}{\cos x}$ [e^xsecx +C]

   c) Prove that for an element 'a' from Boolean algebra, a.a=a.

   d) If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$. $\left[\frac{\log\sin y + y\tan x}{\log\cos x - x\cot y}\right]$

   e) Evaluate $\int_0^{\pi/2} \frac{e^{\sin x}}{e^{\sin x}+e^{\cos x}}dx$ $[e^x \sec x + C]$

   f) Find the equation of the tangent to x^2=12y which is perpendicular to the line 3x+y=0. [3y=x-1]

   g) The probability of a male birth is 0.52. if a woman has three children, what is the probability that at least two are boys? [0.53]

   h) Evaluate $\underset{x\to 0+}{Lt}(1+\sin x)^{\cot x}$. [e]

   i) Express $\frac{1+\cos A + i\sin A}{\sin A + i + i\cos A}$ in A+iB form. [sinA – i cosA]

   j) Solve the differential equation: $xdy - ydx + y^2dx = 0$. [Cy=x(y-1)]

Question 2

a) Using the properties prove that, $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$.

b) If $f(x)=\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ prove that $[f(x)]^{-1}$=f(-x)

Question 3

a) Solve $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$. [1/4]

b) Draw the circuit: $(abc+abc'+ab'c+a'bc)$. Simplify it by the laws of Boolean. Construct the simplified circuit and show that when any two switches are on the lights are on. [ab+bc+ca]

Question 4

a) Find the equation of the ellipse whose foci are at (±2,0) and whose latus rectum is 6. [$3x^2+4y^2=48$]

b) Find the points on $y=x^2-2x$, where the tangent to the curve is parallel to the chord joining (1,-1) and (3,3) by the help of LMVT. [(2,0)]

Question5

a) If $\log y=\cos^{-1}x$, prove that $(1-x^2)y_2-xy_1=y$.

b) A large window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 metres find the dimensions of the rectangle that will produce the largest area of the window. [12/(6-√3),(18-6√3)/(6-√3)]

Question 6

a) There are 3 urns X,Y,Z. which contains 4 red, 3 black and 3 red, 2 black and 2 red, 3 black balls resp. Two ball are drawn from X and one from anyone of Y or Z. Find the probability that atleast one black ball is drawn.[6/7]

b) In a town of 6000 people, 1200 are over 50 yrs old and 2000 are female. It is known that 30% of the female are over 50 years. What is the probability that an individual from the town is either female or over 50 yrs?[13/30]

Question 7

a) Calculate Karl Pearson's coefficient of correlation between Accounts and Mathematics marks:

Accounts	18	40	23	32	27	19	38	40
Mathematics	22	0	17	8	13	21	2	0

[-1, high]

b) In the estimation of regression equation of X and Y the following results were obtained:

$\overline{X}=90, \overline{Y}=70, N=10, \sum(X-\overline{X})^2=6360, \sum(Y-\overline{Y})^2=2860, \sum(X-\overline{X})(Y-\overline{Y})=3900.$

Calculate the value of Y when X=10 by using the suitable regression equation. [21]

Question 8

a) Evaluate $\int \frac{dx}{1-2\sin x}$ $\left[\frac{1}{2\sqrt{2}}\log\left|\frac{\tan x/2-2-\sqrt{3}}{\tan x/2-2+\sqrt{3}}\right|+C\right]$

b) Calculate the area bounded by the curve $y=x-3\sqrt{x}$ and the x axis. [13.5]

Question 9

a) If n be a positive integer, prove that $(1+i)^n+(1-i)^n=2^{\frac{n+2}{2}}\cos\frac{n\pi}{4}$

b) Solve: $\frac{dy}{dx}+y\tan x=2x+x^2\tan x.$ $[y\sec x=x^2\sec x+C].$

SECTION B (ANY TWO)

[2x10 =20 marks]

Question 10

a) Find the cartesian and vector equations of a line which passes through the point (1,2,3) and is parallel to the line $\frac{-x-2}{1}=\frac{y+3}{7}=\frac{2z-6}{3}.$ $\left[\frac{x-1}{-2}=\frac{y-2}{14}=\frac{z-3}{3}, \hat{i}+2\hat{j}+3\hat{k}+\lambda(-2\hat{i}+14\hat{j}+3\hat{k})\right]$

b) Find the plane that meets the coordinate axes in points A, B and C and centroid of ΔABC is (α, β, ¥). [x/α+y/β+z/γ=3]

Question 11

a) Find the area of the triangle whose vertices are given by $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=\hat{i}-\hat{j}-3\hat{k}$ and $\vec{c}=4\hat{i}-3\hat{j}+\hat{k}$. [√41.25]

b) Using vectors prove that sin(A+B)=sinAcosB+sinBcosA.

Question 12

a) The probability that, on joining a professional college, a student will successfully complete the course of studies is 3/5. Determine the probability that out of five students joining (i) none and (ii) at least two will successfully complete the course. [32/3125,2853/3125]

b) A company has two plants to manufacture bicycles. The first and second plants manufacture 60% and 40% bicycles respectively. 80% and 90% of bicycles are rated as standard quality at first and second plants

respectively. A bicycle of standard quality was found. Find the probability that it come from second plant. [0.36]

SECTION C (ANY TWO)

[2x10 =20 marks]

Question 13

a) A bill for Rs 7650 was drawn on 8 March, 2003 at 7 months. It was discounted on 18 May, 2003 and the holder of the bill received Rs 7497. What rate of interest did the banker charge? [5%]

b) Solve the following linear programming problem graphically: Minimize Z=x-5y+20, subject to the constraints $x - y \geq 0, -x + 2y \geq 2, x \geq 3, y \leq 4, x, y \geq 0$. [4,4,4]

Question 14

a) A company set aside a certain sum for a reserve fund on quarterly basis to enable it to pay off a debenture issue of Rs 239000 at the end of 10 years at 8% p.a. [3956.95]

b) The manufacturing cost of an item consists of Rs 900 as overheads, the material cost is Rs 3 per item and the labour cost Rs $x^2/100$ for x items produced. How many items must be produced to have minimum average cost? [300]

Question 15

a) Calculate the price index number:

Commodity	% increase in price	% of consumption
A	81	4
B	16	12
C	10	3
D	52	7

[135]

b) Assuming a five yearly cycle, calculate the trend by the method of moving averages from the following data of industrial failures in a city: Display the actual and trend values on the same graph paper.

Year	No of failures	Year	No of failures
1982	23	1990	9
1983	26	1991	13
1984	28	1992	11
1985	32	1993	14
1986	20	1994	12
1987	12	1995	9
1988	12	1996	3
1989	10	1997	1