MATHEMATICS

(Maximum Marks: 80)

(Time allowed: Three hours)

*(Candidates are allowed additional 15 minutes for **only** reading the paper.*

They must NOT start writing during this time.)

The Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C***

Section A:** Internal choice has been provided in **two questions of two marks each, two questions four marks each and two questions of six marks each.

Section B:** Internal choice has been provided in **one question of two marks and one question of four marks.

Section C:** Internal choice has been provided in **one question of two marks and one question of four marks.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A (65 Marks)

Question 1 **[15×1]**

In sub-parts (i) to (x) choose the correct option and in sub-parts (xi) to (xv), answer the questions as instructed.

(i) The domain of the function $f(x) = \sqrt{9 - x^2}$ is:

a) $\{-3 \leq x \leq 3\}$

b) $\{x \leq -3 \text{ and } x \geq 3\}$

c) $\{x \geq 3\}$

d) $\{-3 \geq x\}$

(ii) If $\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 4$ and $\theta \leq \frac{\pi}{2}$, then θ is:

a) $\frac{\pi}{3}$

b) $\frac{\pi}{6}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{2}$

(iii) If $\theta = 4530°$, then $\sin 4530°$ is:

a) $\frac{1}{2}$

b) $-\frac{1}{2}$

c) $\frac{1}{\sqrt{2}}$

d) $\frac{\sqrt{3}}{2}$

(iv) If third term of G.P is 2, then the product of its first 5 terms is:

a) 32

b) 64

c) 16

d) 128

(v) If α, β are the roots of the equation $x^2 - 2x + 1 = 0$, then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is:

a) 4

b) 1

c) 2

d) 0

(vi) If the coefficient of x^2 and x^3 in the expansion of $(3 + ax)^9$ be same, then the value of a is:

a) $\frac{3}{7}$

b) $\frac{7}{9}$

c) $\frac{9}{7}$

d) $\frac{7}{3}$

(vii) Given that $1, \omega, \omega^2$ are the cube roots of unity, the value of $(3 + 5\omega + 3\omega^2)^3$ is:

a) 6

b) 8

c) 12

d) 16

(viii) The point diametrically opposite to the point $(-3,-4)$ on the circle $x^2+y^2+2x+4y-3=0$ is:

a) $(3,-4)$

b) $(-3,4)$

c) $(1,0)$

d) $(3,4)$

(ix) A straight line that has equal intercepts on axes but opposite in magnitude and passing through the point $(3,2)$ is:

a) $x-y=1$

b) $x+y=1$

c) $-x+y=1$

d) $x+y=-1$

(x) $\lim\limits_{x\to\pi}\frac{\sin x}{x-\pi}$ is equal to:

a) 0

b) 1

c) -1

d) None of the above

(xi) Find the numbers x and y if $(x+3, y-6)=(5,\ 5)$.

(xii) How many 4-digit numbers are there with no digit being repeatd?

(xiii) Find the middle term in the expansion of $\left(\frac{x}{y}+\frac{y}{x}\right)^{10}$.

(xiv) Find the derivative of $f(x)=2x^2+4x+5$ at $x=2$.

(xv) A family has exactly 4 children. Assumimg that each child born is equally likely to be boy or girl, then what is the sample space that the event has exactly one girl child?

Question 2 **[2]**

If A = {1, 2, 3, 4, 5, 6,} B = {2, 4, 5, 6, 8, 9, 10}, find A Δ B.

Question 3 **[2]**

In a class of 35 students , 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football?

Question 4 [2]

Prove that: $\frac{Cos\,2A}{1+Sin2A} = Tan\left(\frac{\pi}{4} - A\right)$

Question 5 [2]

(a) In a Δ ABC, prove that $\frac{Sin\,A}{Sin(A+B)} = \frac{a}{c}$

OR

(b) If $\tan\alpha = -2$, find the value of $\sin\alpha$ (α lies in the II Quadrant).

Question 6 [2]

(a) If $\frac{2+3i}{3-4i} = a + ib$, find the value of a and b.

OR

(b) If α and β are roots of the equation $x^2 + kx + 12 = 0$ and $\alpha - \beta = 1$, find k.

Question 7 [4]

Find the domain and range of : $2 - |x - 4|$

Question 8 [4]

(a) Solve: $sin7x + sin\,4x + sinx = 0\ and\ 0 < x < {}^{\pi}/_{2}$

OR

(b) Prove that $\frac{\cos A + \cos 3A + \cos 5A + \cos 7A}{\sin A + \sin 3A + \sin 5A + \sin 7A} = \cot 4A$

Question 9 [4]

Using Mathematical induction, prove that $10^n + 3.4^{n+2} + 5$ is divisible by 9 for all $n \epsilon N$.

Question 10 [4]

(a) Differentiate the function $\sin(2x - 3)$ by First Principle of differentiation.

OR

(b) Evaluate: $\lim_{x\to 0} \frac{(1-x)^n - 1}{x}$

Question 11 [6]

(a) If 'x' be real, find the maximum and minimum value of: $y = \frac{x+2}{2x^2+3x+6}$

OR

(b) If α, β be the roots $x^2 + lx + m = 0$, then form an equation whose roots are: $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$

Question 12 [6]

The sum of three consecutive numbers of a G.P is 56. If we substract 1, 7 and 21 from these numbers in the order, the resulting numbers form an A.P., find the numbers.

Question 13 [6]

(a) Find the equation of the circle which passes through the points (2, 3), (4, 5) and the centre lies on the line $y - 4x + 3 = 0$.

OR

(b) Find the equation of acute angled bisector of lines:
$3x - 4y + 7 = 0$ and $12x - 5y - 8 = 0$

Question 14 [6]

Find the mean and standard deviation of the following frequency distribution:

Marks	0 – 4	5 –9	10–14	15– 19	20– 24	25– 29	30– 34	35– 39
No. of students	2	5	7	13	21	16	8	3

SECTION B (15 Marks)

Question 15 [5×1]

In sub-parts (i) and (ii) choose the correct option and in sub-parts (iii) to (v), answer the questions as instructed.

(i) In the ellipse $\frac{x^2}{6} + \frac{y^2}{8} = 1$, the value of eccentricity is

a) $\frac{1}{2}$

b) $\frac{2}{3}$

c) $\frac{1}{3}$

d) $\frac{3}{2}$

(ii) If the distance between the points $(a, 2, 1)$ and $(1, -1, 1)$ is 5, then values of a are:

a) $-5, -3$

b) $5, -3$

c) $-5, 3$

d) $5, 3$

(iii) Find the equation of the hyperbola whose foci are $(0, \pm 6)$ and conjugate axis is $2\sqrt{11}$.

(iv) If the line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$, then find the value of m.

(v) Write the converse of the statement: If a number is divisible by 9, then it is divisible by 3.

Question 16 [2]

(a) Construct the truth table for the compound proposition $(\sim p \vee q) \Rightarrow r$

OR

(b) Show that the statement: 'if $x + 3 = 9$, then $x = 6$' is true by method of contrapositive.

Question 17 [4]

(a) Find the equation of ellipse whose focus (1, 2), directrix $3x + 4y - 5 = 0$ and eccentricity is $\frac{1}{2}$

OR

(b) Find the centre, focus, eccentricity and latus rectrum of the hyperbola $16x^2 - 9y^2 = 144$.

Question 18 [4]

In what ratio the point P(–2, y, z) divides the line joining the points A(2, 4, 3) and B(–4, 5, –6). Also, find the coordinates of point P.

SECTION C (15 Marks)

Question 19 [5×1]

In sub-parts (i) and (ii) choose the correct option and in sub-parts (iii) to (v), answer the questions as instructed.

(i) The price index of a commodity is 140. Then, the percentage increase in price of the commodity in current year as compared to the base year is:

a) 4%
b) 40%
c) 20%
d) 10%

(ii) Q_1 is always equal to:

a) P_1
b) P_{10}
c) P_{25}
d) P_{50}

(iii) Find the Q_1 and Q_3 for the following distribution: 5, 3, 6, 3, 13, 9, 8, 24, 19, 20, 18.

(iv) Find D_7 of the distribution x_i: 18, 20, 9, 15, 21, 26, 14, 13, 27, 22, 16, 28.

(v) If a machine costs Rs 10000/- in the year 2005 and Rs 18000/- in year 2008, then find the price relative.

Question 20 [2]

(a) The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys is 70 kg and that of girls in the class is 55 kg. Find the number of boys and girls in the class.

OR

(b) The following table gives weekly consumption of milk (in litres) of 40 families:

Milk in litres	10-14	15-19	20-24	25-29	30-34
No. of families	1	6	8	11	14

Find the 75^{th} percentile.

Question 21 [4]

(a) Find the correlation coefficient r(x, y) if:

$$n = 10, \sum x = 60, \sum y = 60, \sum x^2 = 400, \sum y^2 = 580, \sum xy = 305,$$

OR

(b) Find the Karl Pearson's coefficient of correlation between X and Y for the following data:

X	5	4	3	2	1
Y	4	2	10	8	6

Question 22 [4]

The production of a soft drink company in thousands of litres during each month of a year is as follows:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Production in 1000 of litres	1·2	0·8	1·4	1·6	1.8	2·4	2·6	3·0	3·6	2·8	1·9	3·4

Calculate the five monthly moving averages and show these moving averages on a graph paper.

(NOTE: The total weightage of each Unit covered in the question paper shall be as specified in the syllabus, covering all the chapters of each Unit. The weightage of each question in the Question Paper shall be as indicated in this Specimen Paper. However, the number of MCQs given in Question Nos. 1, 15 and 19 may vary from year to year.)